Scalar Decay Constant and Yukawa Coupling in Walking Gauge Theories

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We propose an approach for the calculation of the yukawa coupling through the scalar decay constant and the chiral condensate in the context of the extended technicolor (ETC). We perform the nonperturbative computation of the yukawa coupling based on the improved ladder Schwinger-Dyson equation. It turns out that the yukawa coupling can be larger or smaller than the standard model (SM) value, depending on the number N_D of the weak doublets for each technicolor (TC) index. It is thus nontrivial whether or not the huge enhancement of the production of the scalar via the gluon fusion takes place even for a walking TC model with a colored techni-fermion. For the typical one-family TC model near conformality, it is found that the yukawa coupling is slightly larger than the SM one, where the expected mass of the scalar bound state is around 500 GeV. In this case, the production cross section via the gluon fusion is considerably enhanced, as naively expected, and hence such a scalar can be discovered/excluded at the early stage of the LHC.

I. INTRODUCTION

The direct searches for the standard model (SM) Higgs boson have been intensively performed at the Tevatron [1] and at the LHC [2, 3]. For these Higgs searches, the significant Higgs production process is the gluon fusion channel. If there are extra colored chiral fermions like in the fourth generation model [4, 5], the Higgs production should be enhanced and thus even a relatively heavy Higgs boson can be surveyed at the early stage of the LHC [6, 7].

The walking technicolor (WTC) is a candidate of the dynamical electroweak symmetry breaking (DEWSB) scenario [8–11]. It can resolve the problems of the flavor changing neutral current (FCNC), too light fermion masses and too light pseudo Nambu-Goldstone (NG) bosons, which were serious difficulties in the QCD-like TC [12, 13]. Although the QCD-like TC was strongly disfavored by the precision measurements [14], the estimate of the S-parameter in the QCD-like TC is not applicable for the WTC. Evidence of the reduction of the S-parameter is reported in the ladder Schwinger-Dyson (SD) and Bethe-Salpeter (BS) approach [15], and also in the lattice simulation [16]. In the holographic WTC model, one can find a parameter space with a small S (~ 0.1) [17, 18].

A "light" scalar, so-called the techni-dilaton (TD), which is the pseudo NG boson associated with the scale symmetry breaking, is predicted in the WTC [9, 19, 20].

The TD mass near the critical point has been suggested as $M_{\rm TD} \sim \sqrt{2}m$ in the context of the gauged Nambu-Jona-Lasinio (NJL) model [21], where m represents the dynamically generated fermion mass. For recent discussions on the TD mass in the criticality limit, see Refs. [22, 23]. The straightforward calculation in the ladder SD and BS approach suggests numerically $M_{\rm TD} \sim 500$ GeV for the typical one-family TC model [24].

It is noticeable that the early stage of the LHC has the sensitivity to such a heavy Higgs [6, 7]. Notice that the gluon fusion process counts the number of the colored particles and also depends on the magnitude of their yukawa couplings. In particular, the estimate of the yukawa coupling is not so trivial in the DEWSB scenario, because a nonperturbative computation is inevitably required.

In this paper, we propose an approach for the calculation of the yukawa coupling through the scalar decay constant and the chiral condensate. We will adopt the ladder SD approach as a nonperturbative method. In principle, these values would be extracted from the lattice simulation.

Let us derive a relation between the scalar decay constant and the yukawa coupling.

Suppose that the extended technicolor (ETC) sector generates the four-fermion interaction,

$$\mathcal{L}_{4F} = G_f \bar{\psi}\psi \bar{f}f, \tag{1}$$

where ψ and f denote the techni and SM fermions. The SM fermion mass m_f is obtained from the techni-fermion condensate,

$$m_f = -G_f Z_m^{-1} \langle \bar{\psi}\psi \rangle_R, \tag{2}$$

with the renormalization constant $Z_m \sim m/\Lambda_{\text{ETC}}$, where Λ_{ETC} is the ETC scale and the subscript R represents the renormalized quantity. The scalar decay constant F_σ for the scalar current is defined by

$$\langle 0|(\bar{\psi}\psi(0))_R|\sigma(q)\rangle \equiv F_\sigma M_\sigma, \qquad (3)$$

where M_σ is the mass of the scalar bound state σ. Eqs. (2) and (3) immediately yield the following expression of the yukawa coupling between σ and f,

$$g_{\sigma ff} = Z_m^{-1} G_f F_\sigma M_\sigma = \frac{m_f}{\frac{-\langle\bar{\psi}\psi\rangle_R}{F_\sigma M_\sigma}}. \qquad (4)$$

For a graphical expression, see Fig. 1. Since the SM yukawa coupling is given by $g_{hff}^{\text{SM}} = m_f/v$ with $v = 246$ GeV, the ratio of the two is

$$\frac{g_{\sigma ff}}{g_{hff}^{\text{SM}}} = \frac{v}{\frac{-\langle\bar{\psi}\psi\rangle_R}{F_\sigma M_\sigma}}. \qquad (5)$$

We can calculate F_σ through the correlation function Π_σ for the scalar operator, which is defined by

$$\mathcal{F}.\mathcal{T}.i\langle 0|(\bar{\psi}\psi(x))_R(\bar{\psi}\psi(0))_R|0\rangle \equiv \Pi_\sigma(q). \qquad (6)$$

The scalar mass M_σ and the scalar decay constant F_σ can be read from the pole and residue of $\Pi_\sigma(q)$, owing to the spectral representation,

$$\Pi_\sigma(q) = \frac{F_\sigma^2 M_\sigma^2}{-q^2 + M_\sigma^2}. \qquad (7)$$

Note that $\Pi_\sigma(0) = F_\sigma^2$ in this normalization. On the other hand, the (renormalized) second derivative of the effective potential at the stationary point corresponds to the inverse of the two point function at the zero momentum,

$$\frac{d^2V}{d\sigma_R^2} = \Pi_\sigma^{-1}(0) = \frac{1}{F_\sigma^2}, \qquad (8)$$

and thereby it holds

$$\sigma_R^2 \frac{d^2V}{d\sigma_R^2} = \left(\frac{-\langle\bar{\psi}\psi\rangle_R}{F_\sigma}\right)^2, \qquad (9)$$

which is closely connected with the yukawa coupling via Eq. (4). We emphasize that this quantity is obviously independent of the renormalization point.

We perform the calculations of F_σ and $\langle\bar{\psi}\psi\rangle_R$ by using the improved ladder SD equation [22]. For a given M_σ, the yukawa coupling is estimated from Eq. (4). We then find the ratio of the yukawa coupling $g_{\sigma ff}/g_{hff}^{\text{SM}} \simeq 1.2$ for the typical one-family TC model with $N_{\text{TC}} = 2$ and $M_\sigma = 500$ GeV under the realistic setup $m/\Lambda_{\text{ETC}} \sim$
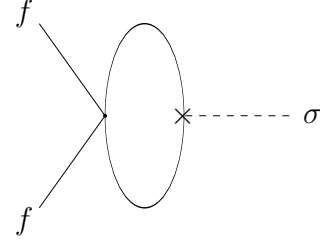

FIG. 1: Yukawa coupling between the SM fermions f and the scalar bound state σ in the framework of the ETC. The technifermion loop generates the mass of f and also intermediates between f and σ.

10^{-3}–10^{-4}. The yukawa coupling was estimated also by using the Ward-Takahashi (WT) identity and a hypothesis of the partially conserved dilaton current (PCDC) [19]. The numerical result via the scalar decay constant agrees with the PCDC approach [22].

In the one-family TC model, the colored technifermions contribute to the production of σ, furthermore. The cross section is thus considerably enhanced. Such a model should be confirmed/excluded at the early stage of the LHC. On the other hand, it is not the case for the model having only one weak doublet and no techni-quark.

This paper is organized as follows: In Sec. II, we study the improved ladder SD equation. An analytical expression for the mass function is derived. In Sec. III, we first review the formalism of the effective potential. By using the analytical expression of the mass function, we calculate F_σ and $\langle\bar{\psi}\psi\rangle_R$. We then obtain the yukawa coupling $g_{\sigma ff}$. Also, the phenomenological implications are briefly discussed. Sec. IV is devoted to summary and discussions.

II. GAP EQUATION

We adopt the ladder SD equation as a nonperturbative approach. In order to incorporate the running effects of the gauge coupling, the improved one has been studied [25]. With the bare mass m_0, it is written by

$$B(x) = m_0 + \int_0^{\Lambda^2} dy \frac{yB(y)}{y + B^2(y)} \frac{\lambda(\max(x,y))}{\max(x,y)}, \qquad (10)$$

where x and y represent the Euclidean momenta, and the normalized gauge coupling $\lambda(x)$ is defined by

$$\lambda(x) \equiv \frac{3C_F \, \alpha(\mu^2 = x)}{4\pi}. \qquad (11)$$

We also introduced the cutoff Λ for the ladder SD equation.

In the two-loop approximation, the renormalization group equation (RGE) of α is [26]

$$\mu^2 \frac{\partial}{\partial \mu^2} \alpha = \beta(\alpha) = -b_0 \alpha^2 - b_1 \alpha^3, \qquad (12)$$

with

$$b_0 = \frac{1}{12\pi}(11C_A - 4N_f T_R), \qquad (13)$$

and

$$b_1 = \frac{1}{24\pi^2}\left[17C_A^2 - 2N_f T_R(5C_A + 3C_F)\right], \qquad (14)$$

where N_f represents the number of flavor and the group theoretical factors are

$$C_A = N_{\mathrm{TC}}, \quad T_R = \frac{1}{2}, \quad C_F = \frac{N_{\mathrm{TC}}^2 - 1}{2N_{\mathrm{TC}}}, \qquad (15)$$

for $SU(N_{\mathrm{TC}})$ gauge theories.

When $b_0 > 0$ and $b_1 < 0$, the Caswell–Banks–Zaks infrared fixed point (CBZ-IRFP) [27] emerges,

$$\alpha_* \equiv \frac{b_0}{-b_1}. \qquad (16)$$

By using the CBZ-IRFP α_* and the Lambert function W [28], which is the inverse of xe^x, we can express $\alpha(x)$ analytically [29],

$$\alpha(x) = \frac{\alpha_*}{1 + W(z(x))}, \qquad (17)$$

where z is defined by

$$z(x) \equiv \frac{1}{e}\left(\frac{x}{\Lambda_{\mathrm{I}}^2}\right)^{b_0 \alpha_*}, \qquad (18)$$

with the intrinsic scale Λ_{I} ($\sim \Lambda_{\mathrm{ETC}}$), which is analogous to the QCD scale Λ_{QCD}.

The integral form (10) can be rewritten by the differential equation with the IR and UV boundary conditions (BC's). Ignoring $x\frac{d\lambda}{dx}(\propto \beta \ll \lambda)$, the differential form is

$$x^2 \frac{d^2}{dx^2}B(x) + 2x\frac{d}{dx}B(x) + \lambda(x)\frac{xB(x)}{x + B^2(x)} = 0, \qquad (19)$$

and the two BC's are

$$(\text{UV-BC}): \quad x\frac{d}{dx}B(x)\bigg|_{x=\Lambda^2} + B(\Lambda^2) = m_0, \qquad (20)$$

$$(\text{IR-BC}): \quad x^2 \frac{d}{dx}B(x)\bigg|_{x \to 0} \to 0. \qquad (21)$$

Let us solve analytically the improved ladder SD equation (19) in the following approximations.

By using the bifurcation method and also the parabolic deformation of the RGE [22],

$$\beta(\alpha) \to -b_0 \alpha(\alpha_* - \alpha), \qquad (22)$$

whose solution is

$$\alpha(x) = \frac{\alpha_*}{1 + e^{-1}\left(\frac{x}{\Lambda_{\mathrm{I}}^2}\right)^{b_0 \alpha_*}}, \qquad (23)$$

we analytically obtain the solution of the linearized ladder SD equation,

$$\frac{B(x)}{B_0} = c_1 \left(\frac{x}{B_0^2}\right)^{-\frac{1-\omega}{2}} F\left(-\frac{1-\omega}{2s}, \frac{1+\omega}{2s}, 1 + \frac{\omega}{s}; 1 - \frac{\lambda_*}{\lambda(x)}\right)$$

$$+ d_1 \left(\frac{x}{B_0^2}\right)^{-\frac{1+\omega}{2}} F\left(-\frac{1+\omega}{2s}, \frac{1-\omega}{2s}, 1 - \frac{\omega}{s}; 1 - \frac{\lambda_*}{\lambda(x)}\right), \qquad (x \geq B_0^2), \qquad (24)$$

where $F(\alpha, \beta, \gamma; z)$ represents the Gauss's hypergeometric function and B_0 is the normalization factor of the mass function defined by $B(x = B_0^2) = B_0$. We also introduced the normalized IR-FP λ_*,

$$\lambda_* \equiv \frac{3C_F \alpha_*}{4\pi}, \qquad (25)$$

the power factor s,

$$s \equiv b_0 \alpha_* > 0, \qquad (26)$$

and

$$\omega \equiv \sqrt{1 - \frac{\lambda_*}{\lambda_{\rm cr}}}, \qquad \lambda_{\rm cr} \equiv \frac{1}{4}. \qquad (27)$$

The integration constants c_1 and d_1 are determined through the IR-BC and the normalization of $B(x)$. In the limit of $B_0 \ll \Lambda_{\rm I}$, we obtain

$$c_1 = \frac{1 + \omega}{2\omega}, \qquad d_1 = -\frac{1 - \omega}{2\omega}. \qquad (28)$$

The UV-BC yields a relation among m_0, B_0, $\Lambda_{\rm I}$ and ω. In the chiral limit $m_0 \to 0$, it turns out that there appears a nontrivial solution $B_0 \to m \neq 0$, only when ω is pure imaginary $\omega = i\tilde{\omega}$,

$$\tilde{\omega} \equiv \sqrt{\frac{\lambda_*}{\lambda_{\rm cr}} - 1} > 0, \quad \text{i.e.,} \quad \lambda_* > \lambda_{\rm cr}. \qquad (29)$$

This approximation *qualitatively* works well. For details, see Ref. [22].

Another linearizing method is to replace the denominator of the last term of Eq. (19) by $x + B_0^2$ [12]. Introducing $\xi \equiv (B_0/\Lambda_{\rm I})^{2s}$ and $B(x)/B_0 \equiv \Sigma^{(0)}(x) + \xi \Sigma^{(1)}(x) + \cdots$, and also expanding $\lambda(x)$ by ξ, we can solve the linearized ladder SD equation owing to the analytic form of $\Sigma^{(0)}(x)$, $\Sigma^{(0)}(x) = F((1 + \omega)/2, (1 - \omega)/2, 2; -x^2/B_0^2)$ [12]. The solution in the region $x \gg B_0^2$ and $|\lambda(x)/\lambda_* - 1| \ll 1$ is similar to the bifurcation solution (24).

For both linearized solutions, we find the behavior of the mass function in the region where the momentum is large in the sense that $x \gg B_0^2$ and the gauge coupling is slowly running, $\lambda(x)/\lambda_* \approx 1$,

$$\frac{B(x)}{B_0} \simeq \frac{A}{\tilde{\omega}} \left(\frac{x}{B_0^2}\right)^{-\frac{1}{2}} \sin\left(\frac{\tilde{\omega}}{2} \ln \frac{x}{B_0^2} + \delta\right), \qquad (30)$$

where A and δ are

$$A = \sqrt{1 + \tilde{\omega}^2}, \quad \delta = \arctan \tilde{\omega}, \qquad (31)$$

for the former approximation, and

$$A = 2|C|, \quad e^{2i\delta} = \frac{C}{C^*}, \quad C \equiv \frac{\Gamma(1 + i\tilde{\omega})}{\Gamma\left(\frac{1+i\tilde{\omega}}{2}\right)\Gamma\left(\frac{3+i\tilde{\omega}}{2}\right)}, \qquad (32)$$

for the latter one. In particular, for the former and the latter, $A \to 1$ and $A \to 4/\pi$ in the limit $\tilde{\omega} \to 0$, respectively. It means that the analytic property of the mass function can be *qualitatively* approximated by Eq. (30), while there are *quantitatively* ambiguities about the values of A and δ. In the next section, we will fix this quantitative uncertainty by using the numerical analysis of the ladder SD equation with the two-loop running coupling [22].

III.   ESTIMATE OF THE YUKAWA COUPLING

In order to calculate F_σ and $g_{\sigma ff}$, we study the effective potential for σ.

Following Ref. [30, 31], we review the formalism on the effective potential.

The generating functional is defined by

$$W[J] \equiv \frac{1}{i} \ln \int [d\psi d\bar{\psi}][\text{gauge}] e^{i \int d^4 x (\mathcal{L} + J\bar{\psi}\psi)}. \qquad (33)$$

and also the effective action is

$$\Gamma[\sigma] \equiv W[J] - \int d^4 x J\sigma, \qquad (34)$$

where

$$\sigma(x) \equiv \bar{\psi}(x)\psi(x). \qquad (35)$$

For constants σ and J, the effective potential is defined by $V = -\Gamma[\sigma]/\int d^4 x$. Noting that

$$\frac{dV(\sigma)}{d\sigma} = J, \qquad (36)$$

the effective potential is formally given by

$$V(\sigma) = \int d\sigma J, \qquad (37)$$

where J should be regarded as a function of σ in this expression.

In the context of the ladder SD equation, it is convenient to use the IR mass B_0 in the expressions of σ and J. We then obtain

$$V(\sigma) = \int dB_0 \frac{d\sigma(B_0)}{dB_0} J(B_0), \qquad (38)$$

where B_0 should be transformed into σ after the integral. Also, the second derivative of the effective potential is

$$\frac{d^2 V}{d\sigma^2} = \frac{dJ}{dB_0} \left(\frac{d\sigma}{dB_0}\right)^{-1}. \qquad (39)$$

Let us explicitly calculate the effective potential and its second derivative. In the following calculation, it is enough to take the cutoff Λ in the ladder SD equation (10) to the ETC scale,

$$\Lambda \to \Lambda_{\rm ETC}. \qquad (40)$$

We consider only the case with $\lambda_* > \lambda_{\rm cr}$.

The effect of the constant source J is obtained by the replacement

$$m_0 \to m_0 - J. \qquad (41)$$

Note that the UV-BC (20) yields

$$\Lambda_{\mathrm{ETC}}^2 B'(\Lambda_{\mathrm{ETC}}^2) + B(\Lambda_{\mathrm{ETC}}^2) = m_0 - J, \qquad (42)$$

where $B'(x) \equiv \frac{dB(x)}{dx}$. The bare chiral condensation is

$$\sigma \equiv \langle \bar{\psi}\psi \rangle = -\frac{N_{\mathrm{TC}}N_f}{4\pi^2} \int_0^{\Lambda_{\mathrm{ETC}}^2} dx\, x \frac{B(x)}{x + B^2(x)} . \qquad (43)$$

By using the ladder SD equation (10), we also find

$$\sigma = \frac{N_{\mathrm{TC}}N_f}{4\pi^2} \frac{\Lambda_{\mathrm{ETC}}^4}{\lambda_{\mathrm{ETC}}} B'(\Lambda_{\mathrm{ETC}}^2), \qquad (44)$$

where $\lambda_{\mathrm{ETC}} \equiv \lambda(\Lambda_{\mathrm{ETC}}^2) \sim \lambda_*$ and we ignored $x\frac{d\lambda(x)}{dx} \propto \beta \ll \lambda$.

We may employ the approximation (30) in the UV region, and hence obtain

$$m_0 - J =$$
$$\frac{A\sqrt{1+\tilde{\omega}^2}}{2\tilde{\omega}} \frac{B_0^2}{\Lambda_{\mathrm{ETC}}} \sin\left(\tilde{\omega} \ln \frac{\Lambda_{\mathrm{ETC}}}{B_0} + \delta + \arctan\tilde{\omega} \right), \qquad (45)$$

and

$$\sigma = -\frac{N_{\mathrm{TC}}N_f}{4\pi^2} \frac{B_0^2 \Lambda_{\mathrm{ETC}}}{\lambda_{\mathrm{ETC}}}$$
$$\frac{A\sqrt{1+\tilde{\omega}^2}}{2\tilde{\omega}} \sin\left(\tilde{\omega} \ln \frac{\Lambda_{\mathrm{ETC}}}{B_0} + \delta - \arctan\tilde{\omega} \right). \qquad (46)$$

The stationary condition $\frac{dV}{d\sigma} = J = 0$ in the chiral limit $m_0 \to 0$ gives the solution of the ladder SD equation, $\tilde{\omega} \ln \Lambda_{\mathrm{ETC}}/B_0 + \delta + \arctan\tilde{\omega} = n\pi,\ (n = 1, 2, 3, \cdots)$. It is known that the zero node solution $B_0 = B_0^{(1)} \equiv m$ corresponds to the true vacuum [12].

We renormalize σ with fixing the zero node solution m. The renormalized quantity at the true vacuum $B_0 = m$ is

$$\sigma_R = Z_m \sigma \to \langle \bar{\psi}\psi \rangle_R = -\frac{N_{\mathrm{TC}}N_f}{4\pi^2} \frac{A}{\lambda_* \sqrt{1+\tilde{\omega}^2}} m^3, \quad (47)$$

where $Z_m \sim m/\Lambda_{\mathrm{ETC}}$ and λ_{ETC} is also renormalized to λ_*. It is straightforward to calculate the vacuum energy, i.e., the value of the effective potential at the true vacuum,

$$V_{\mathrm{sol}} = V|_{B_0=m} = -\frac{N_{\mathrm{TC}}N_f}{4\pi^2} \frac{A^2}{16\lambda_*} m^4 . \qquad (48)$$

Note that in the limit of $\tilde{\omega} \to 0$ ($\lambda_* \to \lambda_{\mathrm{cr}} = 1/4$), Eq. (48) with $A \to 4/\pi$ reproduces the expression of the vacuum energy in Ref. [32]. We also find the renormalized

second derivative of the effective potential at the true vacuum, which corresponds to the inverse of the square of the scalar decay constant,

$$\frac{1}{F_\sigma^2} = \frac{d^2V}{d\sigma_R^2}\bigg|_{B_0 \to m} = \frac{1+\tilde{\omega}^2}{\frac{N_{\mathrm{TC}}N_f}{4\pi^2}(5-\tilde{\omega}^2)} \frac{\lambda_*}{m^2} . \qquad (49)$$

The square of the chiral condensate over the scalar decay constant is then

$$\left(\frac{-\langle \bar{\psi}\psi \rangle_R}{F_\sigma} \right)^2 = \sigma_R^2 \frac{d^2V}{d\sigma_R^2} = \frac{N_{\mathrm{TC}}N_f}{4\pi^2} \frac{A^2}{\lambda_*} \frac{1}{5-\tilde{\omega}^2} m^4 . \qquad (50)$$

We determine the value of A so as to reproduce the vacuum energy in the numerical analysis of the improved ladder SD equation with the two-loop running coupling [22],

$$\langle \theta_\mu^\mu \rangle = 4V_{\mathrm{sol}} \equiv -\frac{N_{\mathrm{TC}}N_f}{2\pi^2} \kappa_V m^4 . \qquad (51)$$

We show the numerical values of κ_V and the determined A in Table I.

The pseudo-scalar decay constant F_π is connected with the weak scale. For the estimate of F_π, it is convenient to employ the Pagels-Stokar formula [33],

$$F_\pi^2 = \frac{N_{\mathrm{TC}}}{4\pi^2} \int_0^{\Lambda_{\mathrm{ETC}}^2} dx\, x \frac{B^2(x) - \frac{x}{4}\frac{dB^2(x)}{dx}}{(x + B^2(x))^2} . \qquad (52)$$

The numerical factor κ_F between m and F_π is defined by

$$v^2 = N_D F_\pi^2 \equiv \frac{N_{\mathrm{TC}}N_D}{4\pi^2} \kappa_F^2 m^2, \qquad (53)$$

where N_D denotes the number of the weak doublets for each TC index. By definition, $N_D \leq N_f/2$. In Table I, we show the values of κ_F calculated in Ref. [22]. Since the normalization of the mass function $B(x = m^2) = m$ yields $\delta = \arcsin(\tilde{\omega}A^{-1})$ in the approximation (30), we can estimate F_π by using Eq. (30) with the values of A in Table I. We found that the differences of κ_F are about 2%, 1% and 1% from top to bottom in Table I, respectively. Although the approximation (30) is inapplicable in the IR region, it practically works well.

We now describe F_σ, $\langle \bar{\psi}\psi \rangle_R$ and $g_{\sigma ff}$ more explicitly. Eqs. (49) and (53) yield

$$\frac{F_\sigma}{v} = \sqrt{\frac{N_f}{N_D}} \sqrt{\frac{5-\tilde{\omega}^2}{(1+\tilde{\omega}^2)\lambda_*}} \frac{1}{\kappa_F}, \qquad (54)$$

the renormalized chiral condensate is

$$\frac{-\langle \bar{\psi}\psi \rangle_R}{v^3} = \frac{N_f}{N_D\sqrt{N_{\mathrm{TC}}N_D}} \frac{4\pi\sqrt{2\kappa_V}}{\kappa_F^3 \sqrt{(1+\tilde{\omega}^2)\lambda_*}}, \qquad (55)$$

λ_*	$\frac{m}{\Lambda_{\text{ETC}}}$	κ_V	κ_F	A	$\sqrt{\frac{N_D}{N_f}}\frac{F_\sigma}{v}$	$\frac{g_{\sigma ff}}{g_{hff}^{\text{SM}}}\frac{v}{N_D M_\sigma}$
0.305	1.12×10^{-3}	0.685	1.38	1.29	2.59	0.142
0.287	1.08×10^{-4}	0.709	1.42	1.28	2.71	0.148
0.258	5.88×10^{-10}	0.756	1.48	1.25	2.93	0.157

TABLE I: Estimates of A, F_σ/v and $g_{\sigma ff}/g_{hff}^{\text{SM}}$ for several values of λ_*. We read the corresponding values of m/Λ_{ETC}, κ_V and κ_F from the numerical analysis of the ladder SD equation with the two-loop running coupling [22].

	$\frac{\sigma(gg \to \sigma)}{\sigma(gg \to h_{\text{SM}})}$	$\text{Br}(\sigma \to WW)$	$\text{Br}(\sigma \to ZZ)$	$\text{Br}(\sigma \to t\bar{t})$	$\text{Br}(\sigma \to gg)$
One-Family model ($N_{\text{TC}} = 2$)	36	30%	14%	52%	4%
Techni-quark ($N_{TQ} = 4, N_D = 1$)	2.2	30%	14%	52%	4%
No Techni-quark ($N_{TQ} = 0, N_D = 4$)	1.2	31%	15%	54%	$\sim 0\%$
No Techni-quark ($N_{TQ} = 0, N_D = 1$)	0.090	31%	15%	54%	$\sim 0\%$

TABLE II: Ratio of the production cross section of σ to that of the SM Higgs and the branching ratios. The mass of σ is fixed to $M_\sigma = 500$ GeV. We took $N_{\text{TC}} = 2$, $N_f = 8$ and $m/\Lambda_{\text{ETC}} = 1.08 \times 10^{-4}$. For comparison with the typical one-family TC model, we just varied the numbers N_{TQ} and N_D in the second, third and fourth rows.

and hence the ratio of the yukawa coupling (5) reads

$$\frac{g_{\sigma ff}}{g_{hff}^{\text{SM}}} = \sqrt{\frac{N_{\text{TC}}}{N_f}} N_D \frac{\kappa_F^2 \sqrt{5 - \tilde{\omega}^2}}{4\pi \sqrt{2\kappa_V}} \frac{M_\sigma}{v} . \quad (56)$$

We show the numerical values of F_σ/v and $g_{\sigma ff}/g_{hff}^{\text{SM}}$ in Table I. The yukawa coupling for the techni-fermions should be almost the same.

We here note that $N_f \simeq 4N_{\text{TC}}$ in the walking gauge theory. For the yukawa coupling in Table I, we have already put $N_f = 4N_{\text{TC}}$. On the other hand, the number N_D is model-dependent; If all flavors have the weak charge like in the typical one-family TC model, $N_D = N_f/2$. The minimum case is $N_D = 1$.

The scalar mass M_σ is closely related to the dynamically generated techni-fermion mass m in the ladder SD approach. The values of m are estimated from Eq. (53). For the typical one-family TC model with $N_{\text{TC}} = 2$, $N_f = 8$ and $N_D = 4$, it reads $m = 390$ GeV, 380 GeV, 370 GeV from top to bottom in Table I. The handy mass formula in the critical limit of the gauged NJL model, $M_\sigma \simeq \sqrt{2}m$ [21], then yields $M_\sigma \simeq 560$ GeV, 540 GeV, 520 GeV, respectively. These are consistent with the estimate in the BS approach, $M_\sigma \sim v\sqrt{17/N_D} \sim 500$ GeV [24]. For a fixed value of the scalar mass, $M_\sigma = 500$ GeV, the ratios of M_σ and m are $M_\sigma/m = 1.3, 1.3, 1.4$, respectively.

In Ref. [19], by using the WT identity and the PCDC relation, $F_{\text{TD}}^2 M_{\text{TD}}^2 = -4\langle\theta_\mu^\mu\rangle$, the yukawa coupling y_{TD} between the TD and the SM fermions was estimated

as $y_{\text{TD}} = (3 - \gamma_m)m_f/F_{\text{TD}}$. Note that in general, the scalar decay constant F_σ discussed in this paper is different from the TD decay constant F_{TD}, which is $\langle 0|\theta_\mu^\mu(0)|TD\rangle = F_{\text{TD}}M_{\text{TD}}^2$. The estimate of v/F_{TD} is 3/5 for the typical one-family TC model with $N_{\text{TC}} = 2$ and $M_{\text{TD}} = 500$ GeV [22]. This yields $y_{\text{TD}}/g_{hff}^{\text{SM}} = 1.2$ and thus numerically agrees with the corresponding result of $g_{\sigma ff}/g_{hff}^{\text{SM}}$.

Let us briefly discuss the phenomenological implications.

As suggested above, we may identify the scalar bound state σ to the TD. We have already shown that the yukawa coupling is different from that in the usual dilaton [34], when the masses of the SM fermions are originated from the four-fermion interactions as in the ETC. On the other hand, it is reasonable to assume the couplings of σ and the weak bosons are $g_{\sigma WW}/g_{hWW}^{\text{SM}} = g_{\sigma ZZ}/g_{hZZ}^{\text{SM}} = v/F_{\text{TD}}$, as usual [34], where $g_{hWW,hZZ}^{\text{SM}}$ are the SM values.

For a typical mass, $M_\sigma = 500$ GeV, we estimate the enhancement factor of the production cross section of σ via the gluon fusion process and the branching ratios, assuming that there are no other light resonances like the techni-pion. The results for the typical one-family TC model and others are shown in Table II.

For the one-family TC model, which contains $N_{TQ} = 2N_{\text{TC}}$ extra quarks (colored techni-fermions), the enhancement factor of the production of σ compared with the SM is huge ~ 36 in the heavy quark limit, even for $N_{\text{TC}} = 2$. Such a model should be confirmed/excluded

at the early stage of the LHC.

Concerning this large enhancement over 10 times, N_{TQ} and also N_D are crucial. As a demonstration, we may just reduce the numbers N_{TQ} and/or N_D. The results are shown in Table II. For the models with $N_{TQ} = 4, N_D = 1$ and $N_{TQ} = 0, N_D = 4$, the production cross section is fairly comparable to the SM one. However, for the model having only one weak doublet of the techni-fermion and no techni-quark, the production cross section gets much smaller.

It might be worthwhile to mention that compared with the SM, the branching ratio to the top-pair is increasing and that to the weak bosons is decreasing.

We have studied the yukawa coupling in the framework of the ETC. However the ETC sector might not produce fully the top quark mass [13, 35]. In such a class of models, the gluon fusion would not be the main production channel. This is out of scope in this paper.

IV. SUMMARY AND DISCUSSIONS

We proposed the alternative approach to estimate the yukawa coupling via the scalar decay constant and the chiral condensate. By using the improved ladder SD approach, we calculated F_σ, $\langle \bar{\psi}\psi \rangle_R$ and $g_{\sigma ff}$.

For the typical one-family TC model with $N_{TC} = 2$, we numerically found $g_{\sigma ff}/g_{hff}^{SM} \simeq 1.2$ under the realistic setup $m/\Lambda_{ETC} \sim 10^{-3}$–$10^{-4}$, where we took $M_\sigma = 500$ GeV. This numerically agrees with that in the PCDC one [19, 22].

The gluon fusion process depends on the number N_{TQ} of the techni-quarks and also the yukawa coupling, which is proportional to the number N_D of the weak doublets

for each TC index through the relation between v^2 and m^2. The result $g_{\sigma ff}/g_{hff}^{SM} \sim \mathcal{O}(1)$ for the one-family TC model near conformality with $M_\sigma \sim 500$ GeV implies that the production cross section of σ is extremely enhanced. This is noticeable, because the early stage of the LHC has the sensitivity to such a "Higgs" boson [6, 7]. On the other hand, in the models with smaller N_{TQ} and/or N_D, such a big enhancement is unlikely to occur. In particular, the production cross section of σ is suppressed in the model having only one weak doublet of the techni-fermion and no techni-quark.

The branching ratios are also changed from the SM ones. The main decay channel is expected to be the top pair, when the mass of σ is above the threshold of $t\bar{t}$.

In this paper, we employed the ladder SD approach. In the holographic WTC model, it is possible to calculate directly the two point function $\Pi_\sigma(q)$, and thus F_σ and also M_σ. The analysis will be performed elsewhere [36].

In passing, we comment that several dynamical models predict existence of a (relatively) heavy Higgs, which can be surveyed at the early stage of the LHC. For example, the top condensate model with extra dimensions predicts successfully the top mass $m_t \simeq 175$ GeV [37]. The Higgs mass is predicted as $m_H \sim 200$–300 GeV. The yukawa coupling is almost the same as the SM one.

The Higgs boson might reveal itself soon at the LHC. The coming few years will be exciting.

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